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Book Review: *Real-Space Renormalization*

Real-Space Renormalization. Edited by T. W. Burkhardt and J. M. J. van Leeuwen. Springer-Verlag, New York, 1982, 214 pp.

Renormalization group applications in statistical mechanics provide considerable insight into the mathematical structure underlying the physics of critical phenomena. In addition, the collection of numerical results, for example, on critical exponents and critical temperatures, continues to grow with the proliferation of techniques for doing the calculations.

Much of the recent activity along this line is in the subset of renormalization group methods known as "real space." These calculations are done in the spirit of Leo Kadanoff's early scaling ideas using block spins. They follow a decade of renormalization calculations done predominantly in momentum space which start with Kenneth Wilson's early work. The popularity of the real-space approach is promoted by its conceptual simplicity and by the natural way in which its methods are suited to computer studies.

The book, *Real-Space Renormalization*, is a collection of articles written by people who have made recent contributions in this field. The fundamentals of the approach are not taught here. For this one should read, for example, the paper by Niemeijer and van Leeuwen in Vol. Six of the Domb and Green series, *Phase Transitions and Critical Phenomena*. The purpose of this book is to acquaint the reader with a variety of current methods and applications, their strengths, weaknesses, principal results, and extensions. The text of each chapter is densely annotated with references that provide background and details.

The first chapter, "Progress and Problems in Real-Space Renormalization," was written by the editors. Results that illustrate the utility of real-space renormalization (RSR) include calculations of critical exponents in one and two dimensions and the description of nonuniversal properties. Weaknesses of the method occur in attempts to calculate critical exponents near three dimensions, where use of the ϵ expansion in momentum space calculations has produced better results; and in applications to systems

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with long-range interactions. Basic difficulties arise from the lack (except in the case of Monte Carlo renormalization) of a system of internal checks on the reliability and accuracy of results, the lack of an expansion parameter for systematically improving accuracy, and the consequent dependence on drastic truncation whose effects are hard to assess. Included in this chapter is a short discussion of Hilhorst's work in exact renormalization, a method in which space-dependent couplings are introduced by the boundary conditions of a finite lattice. There is also a brief account of phenomenological renormalization, which places the model on an infinitely long strip of spins a finite number of spins wide. Critical exponents are obtained, but corrections to scaling are not available by this method.

In "Bond-Moving and Variational Methods in Real-Space Renormalization," by Burkhardt, the subject matter centers on the Migdal–Kadanoff transformation, a procedure which uses bond-moving and decimation, and the Kadanoff lower-bound variational transformation, a method that seeks a greatest lower bound to the free energy. Because of its simplicity the Migdal–Kadanoff transformation has been applied to an impressive list of models. It has been successful in the calculation of some exact phase diagrams, but calculations of the correlation function are inconsistent with widely accepted scaling behavior, and results for critical exponents are inexact.

In the procedures talked about by R. H. Swendsen in "Monte Carlo Renormalization," the computer is used to generate configurations that are statistically appropriate to the initial Hamiltonian. A RG transformation divides these configurations into blocks. The renormalization equations for the coupling constants are obtained from correlated functions determined by averaging over the configurations. "Large-cell renormalization," in which the entire system is reduced to two blocks, converges slowly. The method is being extended to percolation and polymer problems. For a second method, called "MCRG," which uses small blocks, tables are included which show how estimates of critical exponents are affected by the number of RG iterations, the number of coupling constants included, and the lattice size, in the example of the Ising model in two dimensions.

"The Real Space Dynamic Renormalization Group," by G. F. Mazenko and O. T. Valls, contains an analysis of the two-dimensional kinetic Ising model with spins on a square lattice. Both the statics and dynamics of the system are described by the behavior of the Fourier–Laplace transform of the time- and space-dependent correlation function of the spins. A perturbation expansion is used in which the coupling between cells of spins is the small parameter. Included are several plots describing very well the susceptibility, space-dependent correlation function, and the time- and space-dependent correlation function. Important features of the time- and space-dependent correlation function are displayed in various plots in

which the temperature extends well above the critical temperature, T_c, and into a small region below T_c. Better results for temperatures less than T_c are promised in a later publication by the same authors.

Calculations in surface physics are discussed in "Application of the Real-Space Renormalization to Adsorbed Systems," by M. Schick. Because adsorbate atoms tend to locate at adsorption sites RSRG methods are well suited to such problems. Preservation of the lowest symmetry of the ordered phase is believed to be essential in any renormalization calculation. Two methods which accomplish this are included in this chapter. The sublattice method is illustrated by the example of a lattice consisting of three sublattices. Repulsive nearest-neighbor interaction dominates. The calculations are compared with data from helium adsorbed on graphite; good fits are achieved for temperatures less than T_c. The prefacing method with the introduction of vacancies is useful when the underlying symmetry of the order–disorder transition is the same as that of a more easily analyzed system. For example, the three-state Potts model can be substituted for the triangular lattice gas with repulsive nearest-neighbor interactions. When vacancies are introduced the Potts model becomes the Potts model lattice gas. Good agreement with krypton on graphite experiments is achieved. Included is a phase diagram showing the critical line, tricritical point, and first-order transition. Also mentioned briefly in this chapter is the use of the vacancy in cell mapping and the relation of critical exponents of the Potts model to those of the eight-vertex model, to find exact thermal and magnetic exponents for the q-state Potts model.

The chapter "Renormalization for Quantum Systems" is authored by P. Pfeuty, R. Jullien, and K. A. Penson. Quantum phase transitions are of interest because quantum fluctuations need to be taken into account at low temperatures. Further interest arises from the possibility of transforming statistical mechanical systems in d dimensions to a quantum-mechanical system on a lattice of $d-1$ space dimensions and one time dimension. A method is discussed in which the lattice is divided into blocks of sites. The Hamiltonian for the system is considered to be the sum of a block Hamiltonian and a portion which is due to the coupling between blocks. Eigenstates of the block Hamiltonian (usually low-lying states) are selected as basis states for a new Hamiltonian for the lattice of blocks, with the coupling between blocks treated perturbatively. The recursion equations which evolve for the parameters are used to determine physical quantities such as the ground state energy and the energy gap. The correlation functions for the spins and, near the fixed point, the critical exponents, can be calculated. The number of energy levels included, the number of sites per block, and the order of perturbation can, to some extent, affect the accuracy of the calculation.

H. E. Stanley, P. J. Reynolds, S. Redner, and F. Family are the

authors of "Position-Space Renormalization Group for Models of Linear Polymers, Branched Polymers, and Gels." In this chapter there is a departure from the language of thermodynamics. The reference here is to site and bond probabilities or weights. The first step is to model the system by a lattice of appropriate dimensionality and coordination number to accommodate the number of active sites on the monomer. Vertices of the lattice represent sites; lines represent bonds. To renormalize, a configuration of bonds or sites having the symmetry of the lattice is called a renormalized bond or site, respectively, provided a specified connectivity rule is satisfied by a cluster of bonded sites traversing it. Universality classes are defined by critical exponents such as that of the large cluster diameter exponent, ν. Singularities occur at the value of a weight or probability for which a single cluster spans the entire lattice. When the model requires a pair of independent parameters, there is a critical surface on which crossover behavior can occur. Flow diagrams demonstrating such effects are included.

Finally, it should be mentioned that the index is rich in key words used in the book, a feature which adds greatly to the book's usefulness as a reference.

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